Research Underpinnings, Design Research, and Math Recovery®

Pamela D. Tabor, Ph.D.

The Institute of Education Sciences (IES) of the U.S. Department of Education and the National Science Foundation have developed a set of guidelines that articulate six types of educational research (*Common guidelines for education research and development*, 2013). These types are: 1) Foundational Research, 2) Early-Stage or Exploratory Research, 3) Design and Development Research, 4) Efficacy Research, 5) Effectiveness Research, and 6) Scale-up Research. Too often, the first three types of research are under reported. This purpose of the paper is to highlight these critical types of research that underpin the Math Recovery® intervention and professional development efforts.

Math Recovery® is the outcome of design and development research building on a tremendous body of foundational and exploratory research spanning over nearly a century. Robert Wright, the developer of Math Recovery®, initially built on the philosophical foundation research of Piaget (Piaget, 1941, 1971), Vygotsky (Vygotsky, 1978, 1987), and von Glasersfeld (von Glasersfeld, 1981, 1982, 1983a, 1983b, 1987, 1989, 1995; von Glasersfeld & Kelley, 1982).

Wright drew heavily on the research of Steffe and his colleagues who conducted a series of constructivist teaching experiments (Steffe, 1991a) and initially identified the developmental stages in early arithmetical learning (Cobb & Steffe, 1983; Steffe, 1991b, 1992a, 1992b, 1994, 1995, 2004; Steffe, Cobb, & von Glasersfeld, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983). The research of Cobb and his colleagues: Bausersfeld, Wheatley, Gravemeijer, Yackel, McClain, Wood, and Whitenack (Cobb & Bauersfeld, 1995; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Cobb, McClain, Whitenack, & Estes, 1995; Cobb & Wheatley, 1988; Cobb, Yackel, & McClain, 2000; McClain & Cobb, 1999; Yackel, 2001) along with the research at the Freudenthal Institute in The Netherlands (Anghileri, 2000, 2001; Beishuizen, 1993, 1995, 1999, 2001; Beishuizen & Anghileri, 1998; Blöte, Klein, & Beishuizen, 2000; Gravemeijer, 1994; Menne, 2001; Streefland, 1991; Treffers & Beishuizen, 1999; van den Heuvel-Panhuizen, 1996, 2001a, 2001b, 2001c) further contributed to the development of the framework in the areas of conceptual place value and structuring number. Ian Thompson's (Thompson, 1997, 1999, 2003) research on multi-digit addition and subtraction and the use of algorithms further informed the development of the portion of the framework involving conceptual place value. Dowker's research in effective means of early intervention (Dowker, 1998, 2003, 2004, 2005a, 2005b) and Wright's own early research (Wright, 1989, 1991a, 1991b, 1991c, 1994a, 1994b, 1994c, 1996, 1998a, 1998b) influenced the method of intervention delivery.

Wright incorporated research-based strategies and techniques for mathematics pedagogy from an array of researchers into a coherent, intensive intervention for young children struggling in mathematics. The intervention was patterned after the highly effective (May et al., 2014; May et al., 2013; May, Sirinides, Gray, & Goldsworthy, 2016) early individualized reading intervention, Reading Recovery, developed by Marie Clay. Wright developed frameworks that a) model students' development of early number concepts (stages of early arithmetical learning (SEAL)), base-ten arithmetical strategies, number words and numerals, structuring number 1-20, and early multiplication and division) and b) guide the teaching of early number in these number concepts. He also developed videotaped interview-based assessments

(VIBA), student profile protocols, and an extensive collection of exemplar teaching procedures to advance students' arithmetical development.

Wright received a design and development collaborative research grant of $257,000 from the Australian Research Council (grant code: AM9180064, 1992-1994) with additional funding of $460,000 from regional government and the Catholic school systems that led to the development of Math Recovery®. The outcome of this research was the development of the Mathematics Recovery intervention for first graders and the publication of the first two books in the Math Recovery® series by SAGE, Pub.: *Early numeracy: Assessment for teaching & intervention* (Wright, Martland, & Stafford, 2006) and *Teaching number: Advancing children's skills & strategies* (Wright, Martland, Stafford, & Stanger, 2006), both now in their second edition. Wright and his colleagues have published several research articles and book chapters detailing the methods and results of these studies (Wright, 1998a, 1998b, 2000, 2002, 2003, 2008a, 2008b, 2013; Wright, Cowper, Stafford, Stanger, & Stewart, 1994; Wright, Stanger, Cowper, & Dyson, 1996; Wright, Stewart, Stafford, & Cain, 1998). One novel aspect of the intervention was the use of videotaped interview-based assessments (VIBA) (Ellemor-Collins & Wright, 2008) and the routine use of video recording of all instruction. This practice allows researchers to review all aspects of the intervention for in-depth analysis, but also affords the teachers the opportunity to reflect deeply and share episodes of interest with colleagues and parents. Each teacher develops a library of video clips that are shared at collegial video analysis meetings. These on-going professional development meetings allow teachers to seek advice with puzzling situations and celebrate successes. Selected clips also become part of future professional development courses for new teachers.

The one-on-one, research-developed intervention frameworks and strategies were then adapted for use with classroom teachers in the Australian state of New South Wales in the mid 1990's and later in New Zealand. The *Count Me In Too* and *Count Me In Too, Indigenous* projects (Bobis, 1996, 1997, 1999, 2001a, 2001b, 2003, 2004, 2006, 2009; Bobis et al., 2005; *Count Me In Too: Professional Development Package*, 1998; Gould, 2000; Mitchelmore & White, 2002, 2003; Thomas, Tagg, & Ward, 2002; Thomas & Ward, 2001; Wright & Gould, 2002; Wright, Mulligan, Bobis, & Stewart, 1996), along with related classroom teaching experiments research (Tabor, 2008) form the research basis of the third book in the Math Recovery® series: *Teaching Number in the Classroom with 4-8 year olds, 2nd edition* (Wright, Stanger, Stafford, & Martland, 2015), which is the textbook of the professional development course Add+Vantage MR® Course 1.

Wright was awarded an additional $210,000, three-year research linkage grant from the Australian Research Council (LP0348932) to partner with *Catholic Education Melbourne* with the goal of developing an intervention for low-attaining students in the middle years which expanded the frameworks for numeracy assessment and instruction for students through the fourth grade (Ellemor-Collins & Wright, 2007, 2008, 2009a, 2009b; Ellemor-Collins, Wright, & Lewis, 2007; Wright, 2001; Wright et al., 2008; Wright, Ellemor-Collins, & Lewis, 2007). This research led to the publication of the fourth book in the Math Recovery® series, *Developing number knowledge: Assessment, teaching & intervention with 7—11-year-olds* (Wright, Ellemor-Collins, & Tabor, 2012), the textbook for the intermediate classroom teacher professional development adaptation of Math Recovery®, Add+VantageMR® Course 2.

The most recent addition to the Math Recovery® series is the Add+VantageMR® Fractions Course, developed in part by two research grants from the National Science Foundation: DRL 1252575 awarded to Amy Hackenberg of Indiana University and DRL 1118571 awarded to Evans, Deater-Deckard, Balci,

Norton, and Chang of Virginia Tech. This design research resulted in the creation of a model for the development of fractions concepts, two fractions assessments, an extensive collection of exemplar teaching procedures including a software application designed to advance students' fraction knowledge, (Hackenberg, 2007; Hackenberg & Tillema, 2009; Norton, 2008; Norton et al., 2015; Norton & Wilkins, 2013; Norton et al., 2014), and the publication of the fifth book in the Math Recovery® series: *Developing Fractions Knowledge* (Hackenberg, Norton, & Wright, 2016).

On-going program implementation and professional development funded by the Catholic Education Melbourne has enabled Wright and his research colleague Ellemor-Collins to expand the Learning Framework in Number and develop additional pedagogical tools for assessment and instruction as a part of the Number Intervention F-4 Project (Dineen, 2014; Ellemor-Collins, 2018; Ellemor-Collins & Wright, 2011a, 2011b; Ellemor-Collins, Wright, & McEvoy, 2013; Rumiati, 2010, 2017; Thi, 2016; Wright, 2013). The sixth book in the Math Recovery® series, *The Learning Framework in Number: Pedagogical tools for assessment and instruction* (Wright & Ellemor-Collins, 2018) which communicates these research outcomes will be available in 2018. A direct outcome of these efforts was the expansion of the intervention into the intermediate grades. The USMRC announced the Math Recovery® Specialist (MRSp) courses on March 6, 2018. The second course in this series will expand the Specialist's expertise to offer intervention to struggling intermediate students. Math Recovery® Specialist, Part 2: Intervention and Specialization in Number Learning for the Intermediate Grades (MRSp2) will be available August 2019.

Math Recovery® intervention and professional development courses were developed within a strong tradition of research and from research funded in part by two national governments. The developers continue their design and development efforts. As new relevant research becomes available, the professional development is adapted to incorporate this new knowledge.

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